# X-Cipher: Achieving Data Resiliency in Homomorphic Ciphertexts

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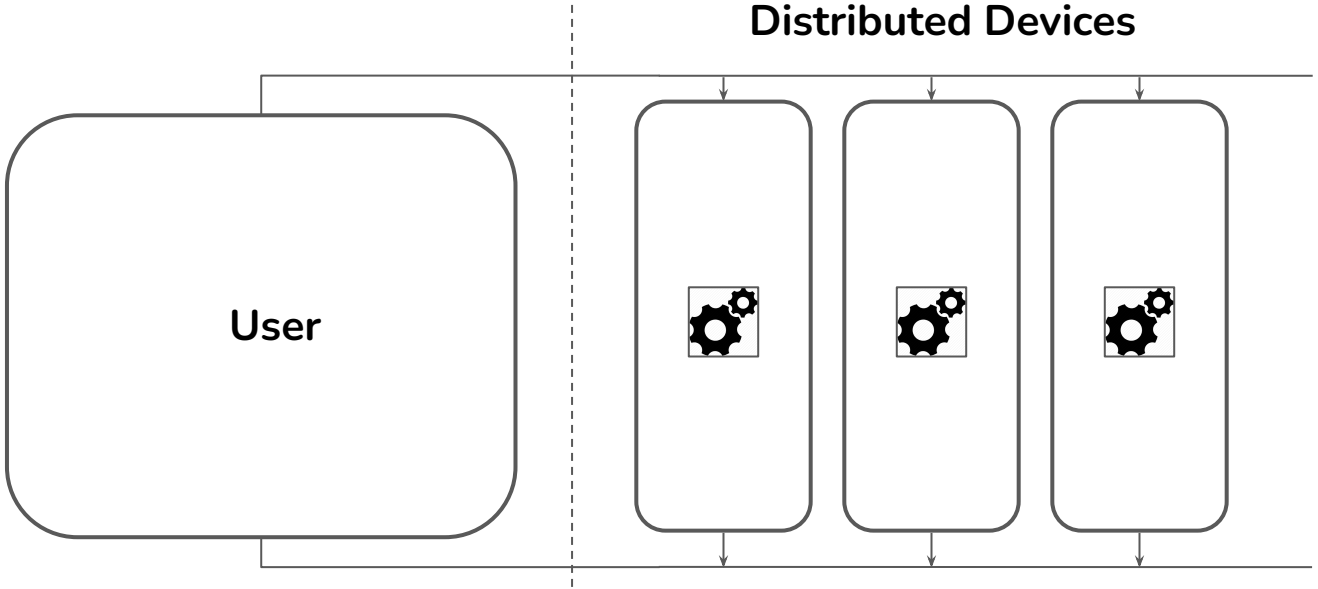
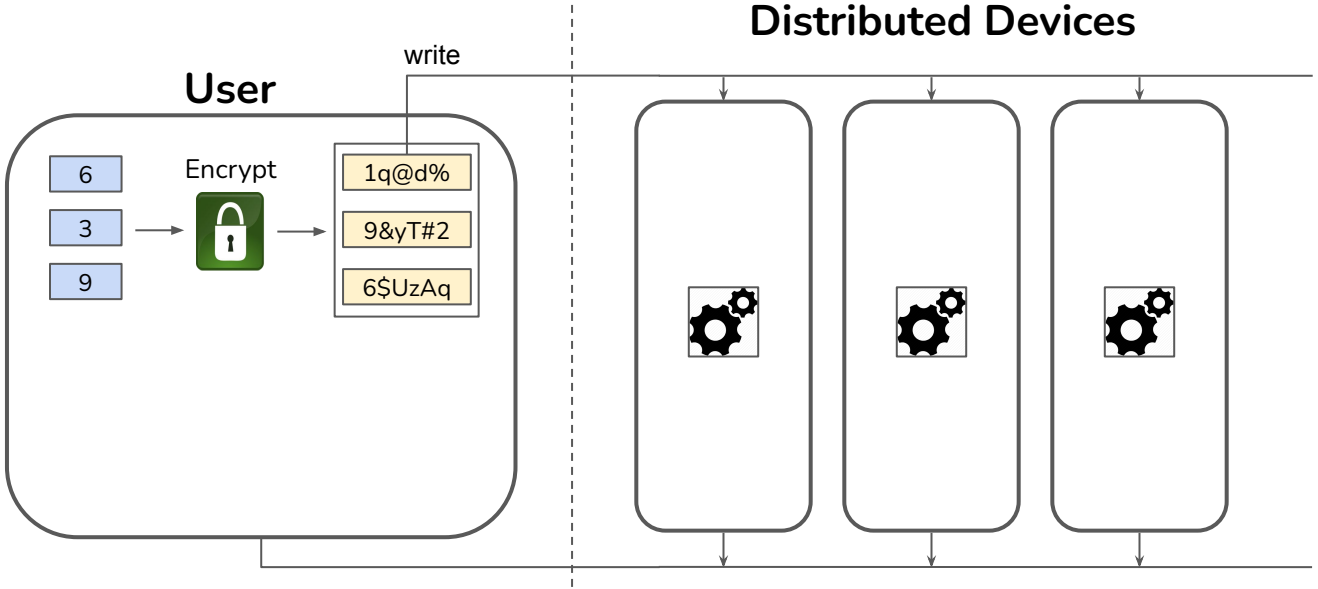
# Computing on the Cloud

- Outsourcing computations to the cloud has become incredibly common



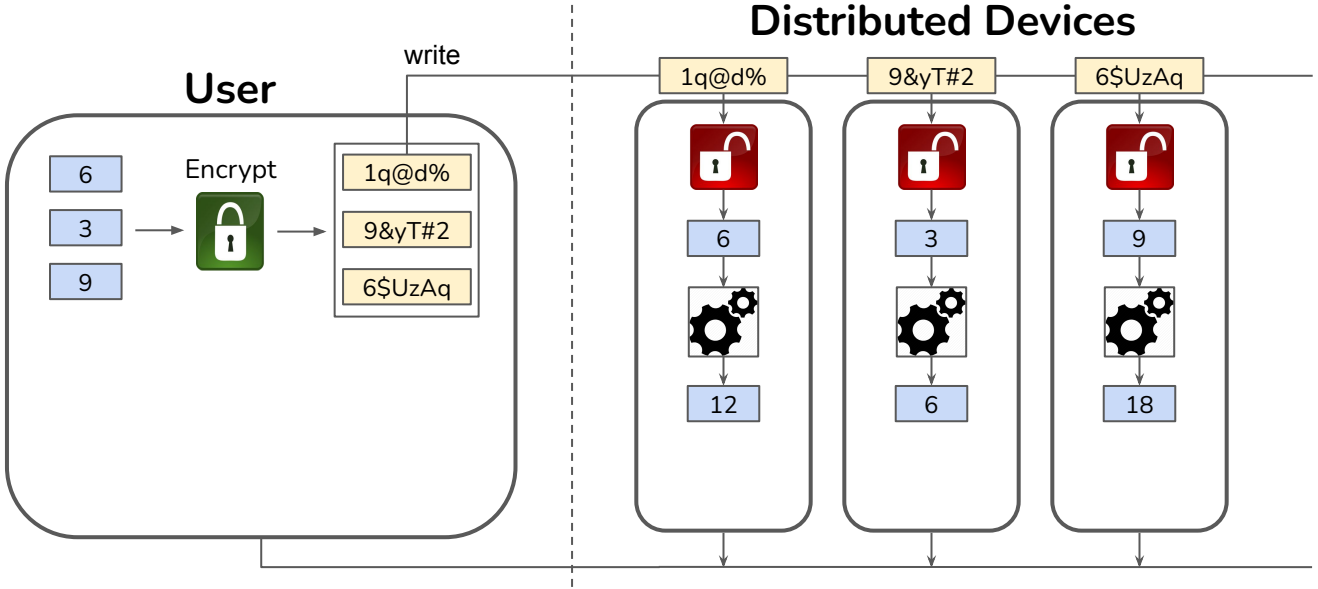
# Computing on the Cloud

- To leverage cloud services on secrets, User's might encrypt their data in-transit



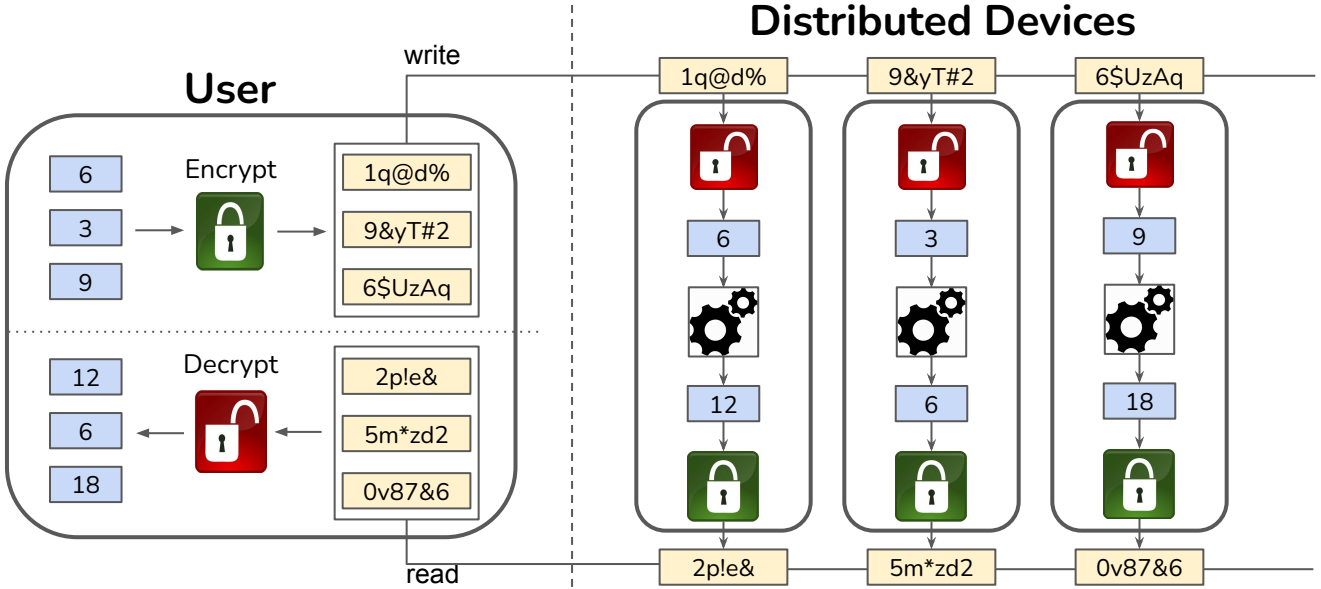
# Computing on the Cloud

- Using a shared shared key, the cloud would decrypt the secrets to compute their outputs



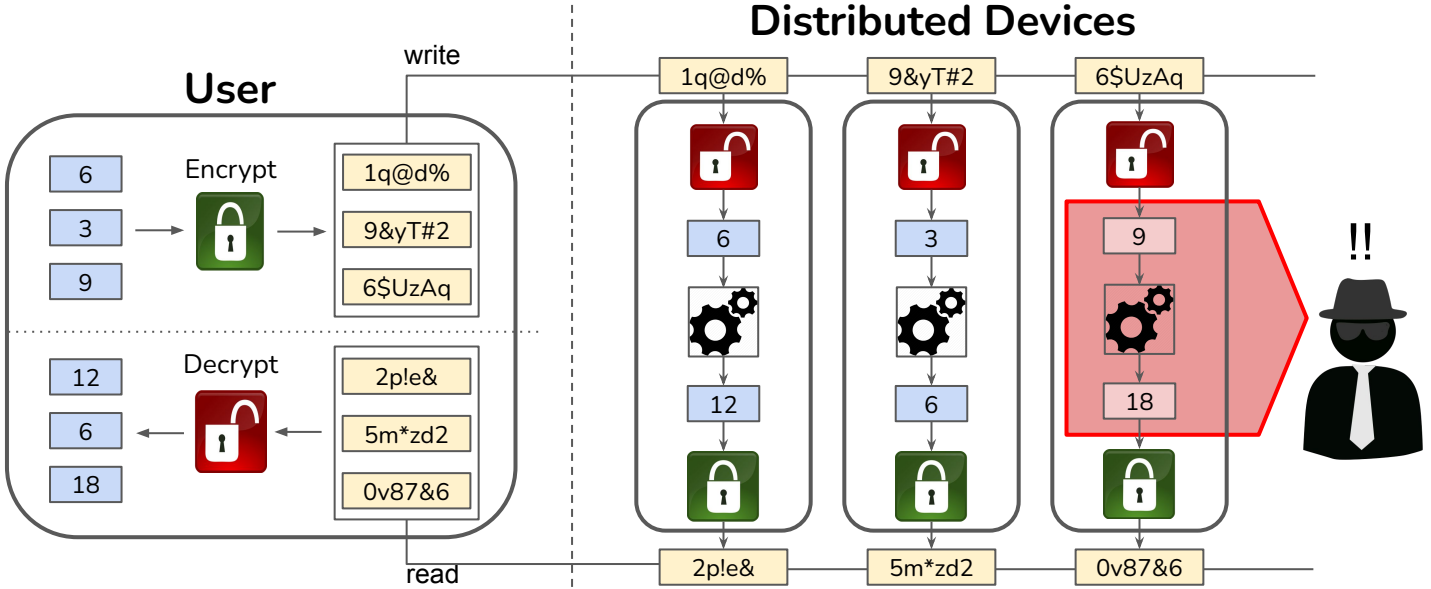
# Computing on the Cloud

- Servers transmit the encrypted output back to the user



# Computing on the Cloud

- Since data is decrypted during computations, could lead to potential leakages



# A solution: Homomorphic Encryption (HE)

- Produces ciphertexts that can undergo computations without decryption
- Protect data while in-storage, in-transit, and ***in-use***

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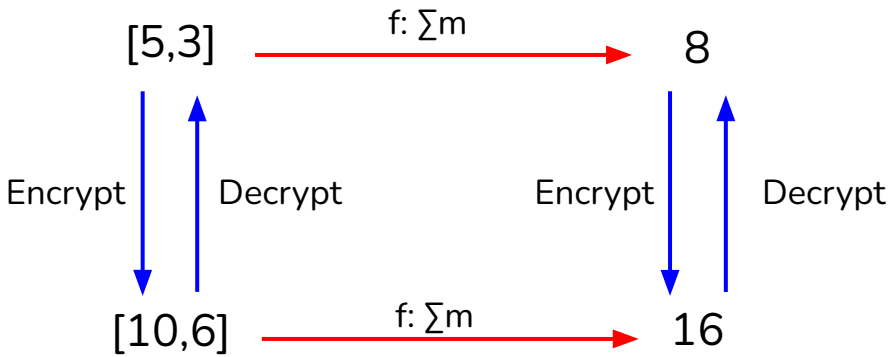
Toy Example:

Encrypt(m) = 2 * m  Decrypt(c) = c / 2

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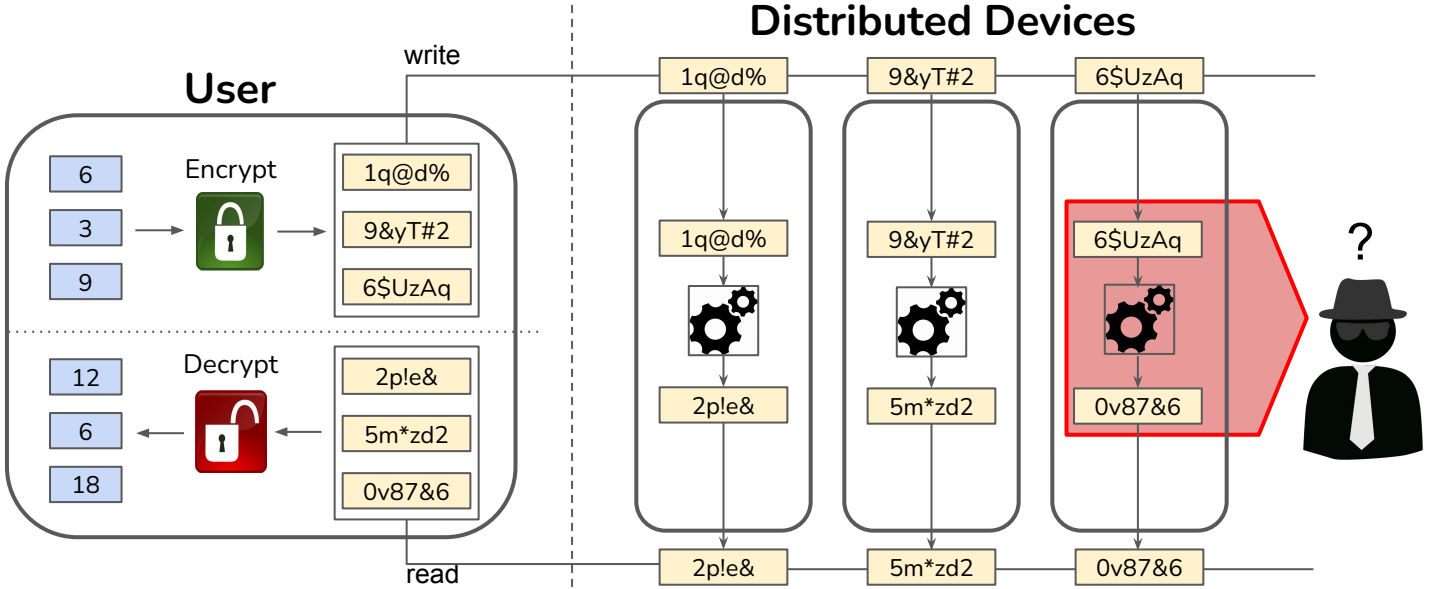
- Partially (add. *or* mult.) or Fully (add. *and* mult.) homomorphic

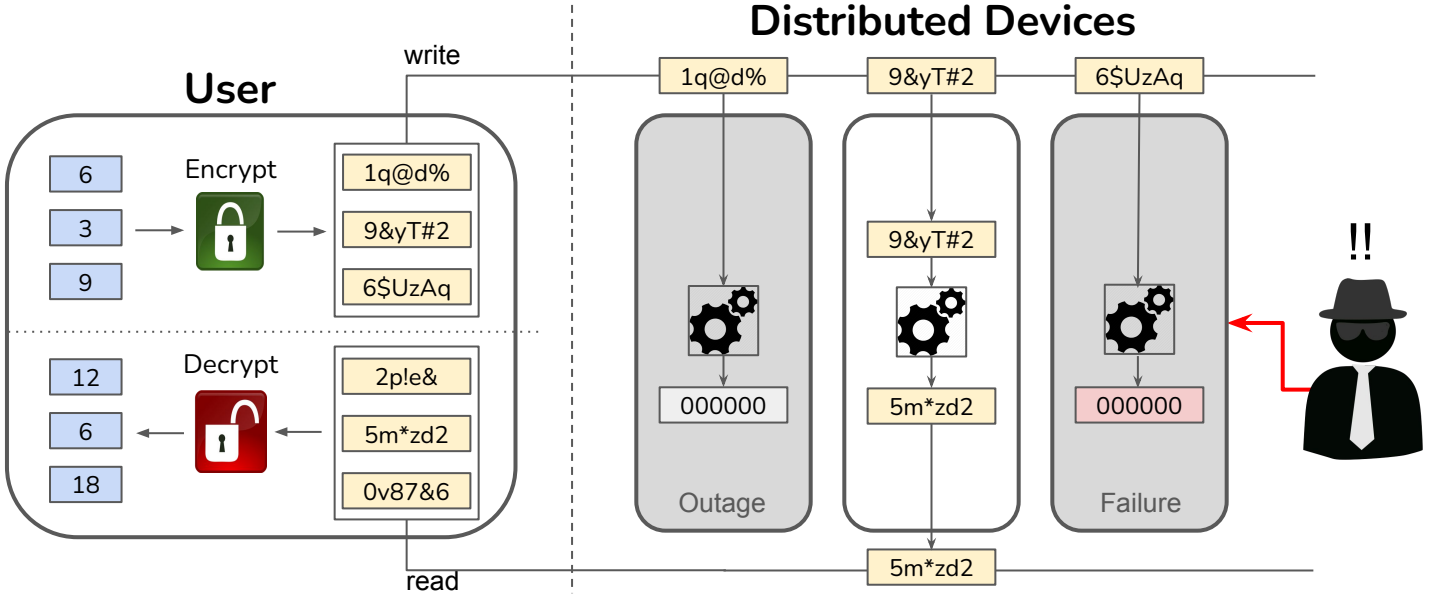
# Computing on the Cloud

- With support for HE, computations can take place without ever revealing the secrets



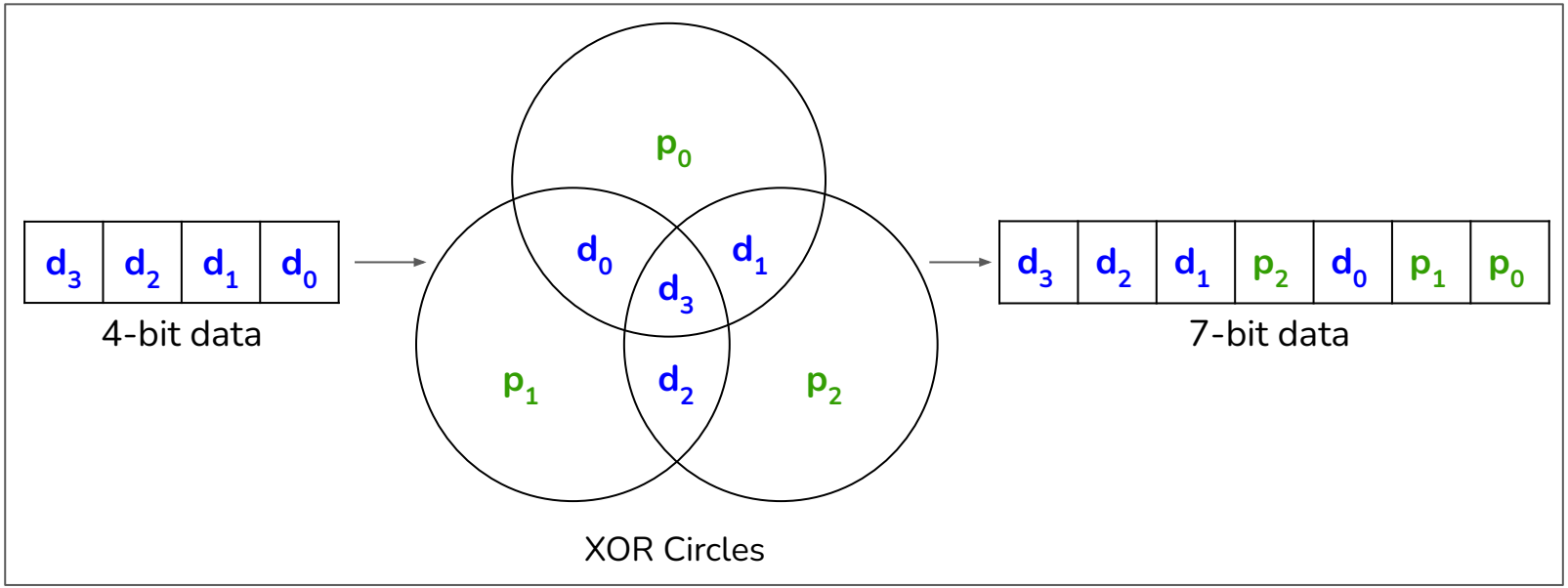
# Challenge: Making HE ciphertexts resilient

- How can we handle data losses in the cloud?



# Common approach: Erasure codes

- Compute erasure codes over plaintext to enable recovery




- Distribute redundancy codewords over servers

# The problem with erasure codes and HE ciphertexts

How can it be applied to ciphertexts?

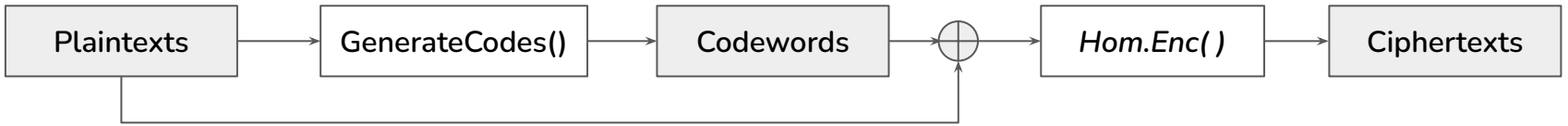
- If trivially applied, would be computed on the ciphertext level


- Problem:
  - Storage impact of codewords is proportional to input
  - HE ciphertexts can be LARGE → Ciphertexts can be large polynomials (Ring-LWE based FHE)
    - ~6000% size increase from plaintext

# Alternative: Codewords first, then encrypt

A version *Encrypting-with-Redundancy*

- Not well studied for Homomorphic Encryption




- Problem:
  - Assume the ciphertexts are not operated on, or only supported for partial homomorphic operations
  - Not applicable for fully homomorphic schemes
  - Not applicable for variety of complex cloud computing operations

# To bridge this gap: X-Cipher

- Enables recovery of fully homomorphic ciphertexts without decryption
- Leverages encoding and packing techniques for optimized storage
- Maintains privacy and recoverability across fully-homomorphic computations

# Fully Homomorphic Encryption (FHE)

- Enables additions and multiplications without decryption
- This work uses schemes based on Ring Learning-with-Errors (Ring-LWE)
  - Elements are based on polynomial ring $R_q = Z_q[x]/\Phi[x]$
  - Plaintext values are encoded into polynomials

Example:

$$A \rightarrow A(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$$

- Compared to standard encryption, FHE has large storage requirement

# Optimized polynomial encoding: Ciphertext Packing

Subfield packing: Packing values into subfields using Chinese Remainder Theorem (CRT)

Polynomial modulus: $\Phi(x) = x^4 + 1 = (x - 2)(x - 2^3)(x - 2^5)(x - 2^7) \pmod{17}$

Each vector element corresponds to a 0 degree polynomial:

**v = [8, 5, 16, 9]**

$1 + x + 7x^2 + 12x^3 \equiv \mathbf{8} \pmod{17, x-2}$

$1 + x + 7x^2 + 12x^3 \equiv \mathbf{5} \pmod{17, x-2^3}$

$1 + x + 7x^2 + 12x^3 \equiv \mathbf{16} \pmod{17, x-2^5}$

$1 + x + 7x^2 + 12x^3 \equiv \mathbf{9} \pmod{17, x-2^7}$

Utilized in CKKS and **BGV** — X-Cipher leverages BGV scheme

Enables SIMD-like homomorphic operations

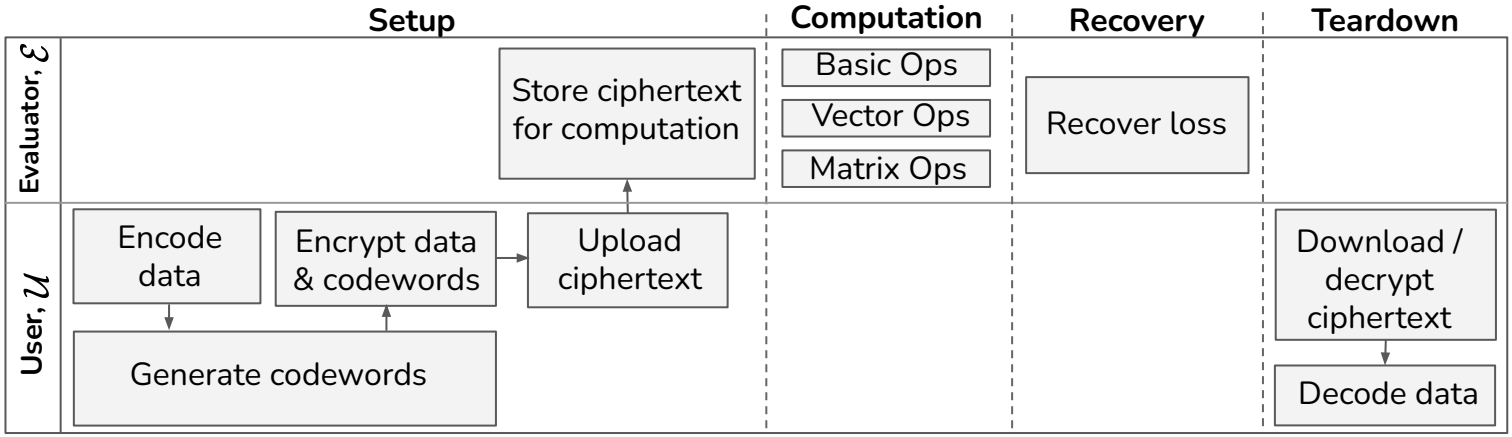
# X-Cipher key ideas:

- Generate code words for plaintext vectors and pack them alongside each other for optimized storage and recovery capability

- Provide homomorphic recovery algorithms to enable data recovery without requiring decryption

- Enable computations over homomorphic ciphertexts that maintain the ability to recover intermediate or final results

# X-Cipher Workflow



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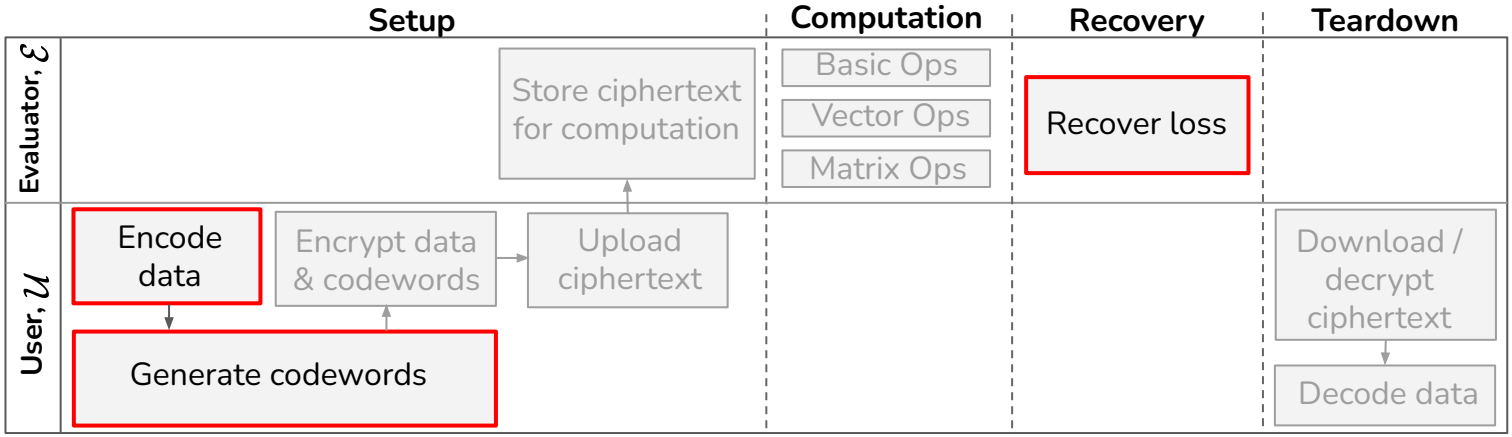
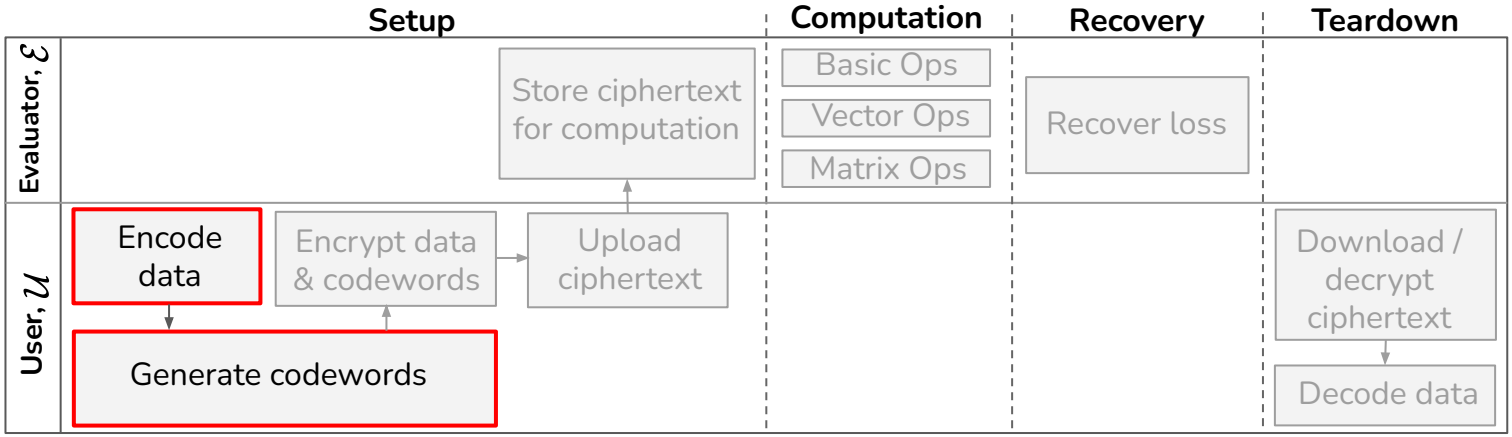
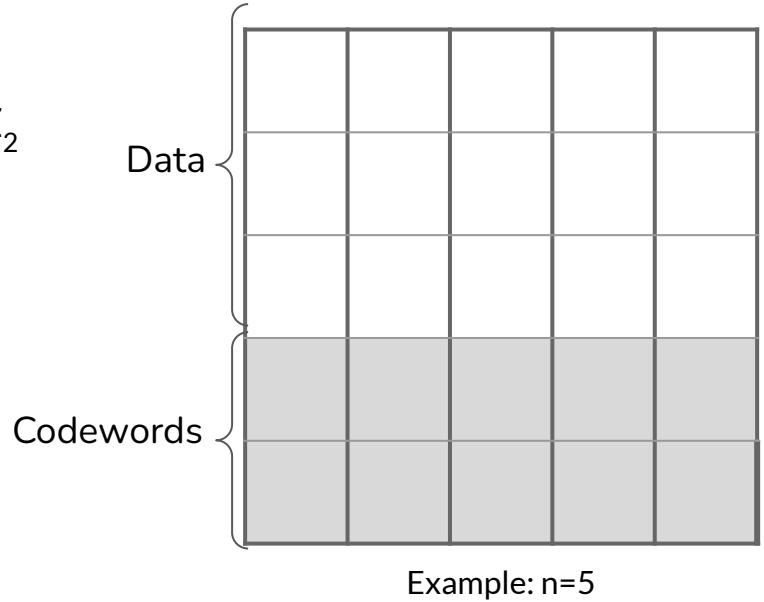


# X-Cipher Workflow



# X-Code for codewords

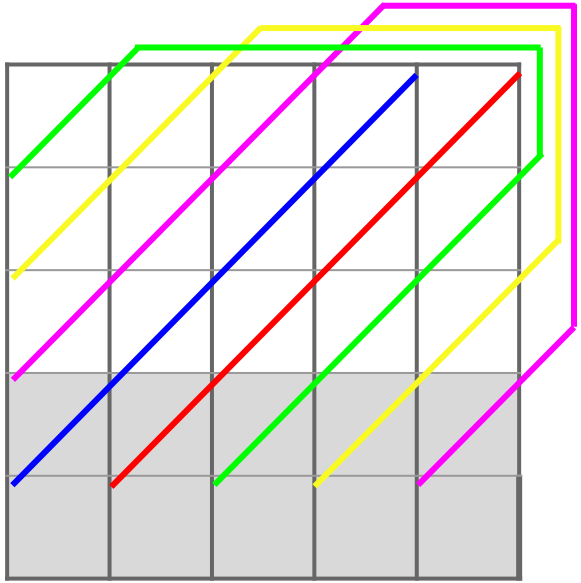
- Utilize X-Code erasure codes
  - Dual parity → two column recovery
  - Only requires xor operations → Addition in $Z_2$
- Construct an *n x n* grid
  - First *n-2* rows are **data**
  - Last 2 rows with **codewords**




Example: n=5

# How to generate codewords?

- First row of codewords is computed by addition along slope +1 diagonals

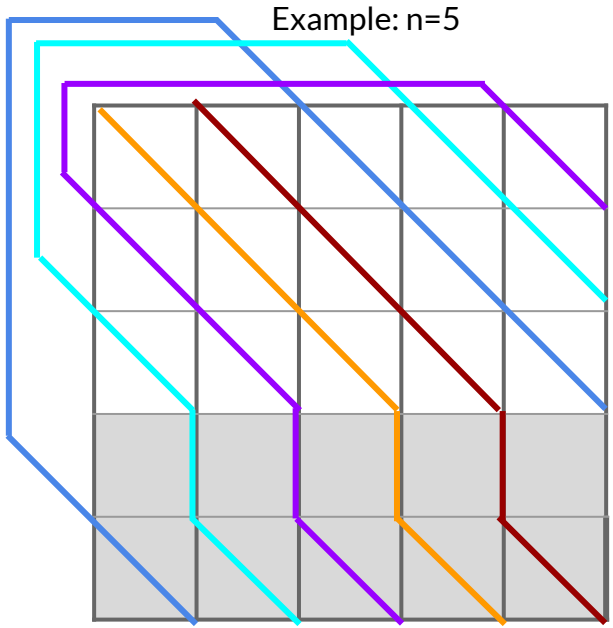


Example: n=5

# How to generate codewords

- First row of codewords is computed by addition along +1 slope diagonals

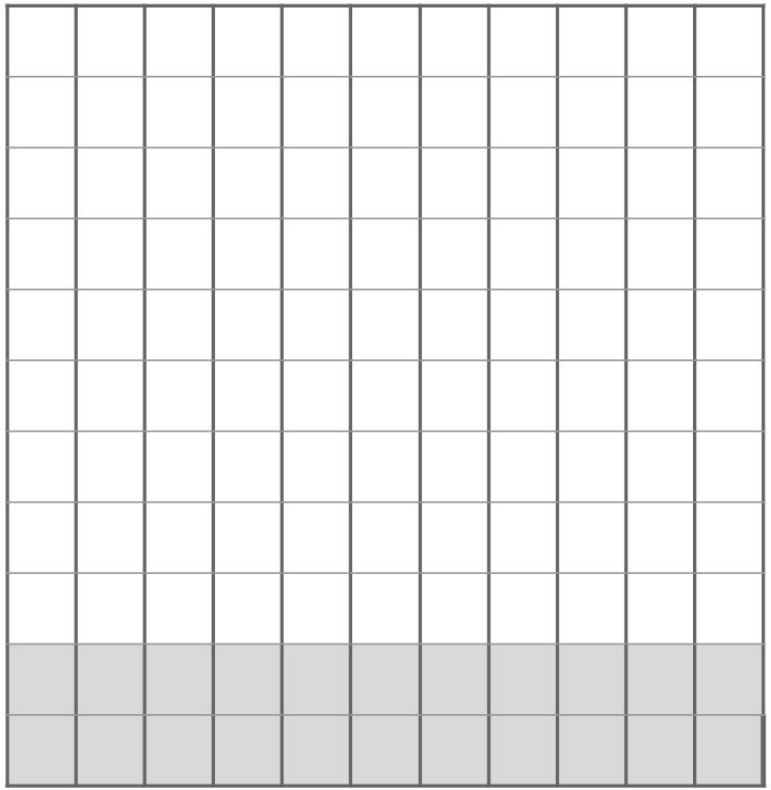
- Second row of codewords is computed by addition along -1 slope diagonals



# X-Code structure for large data

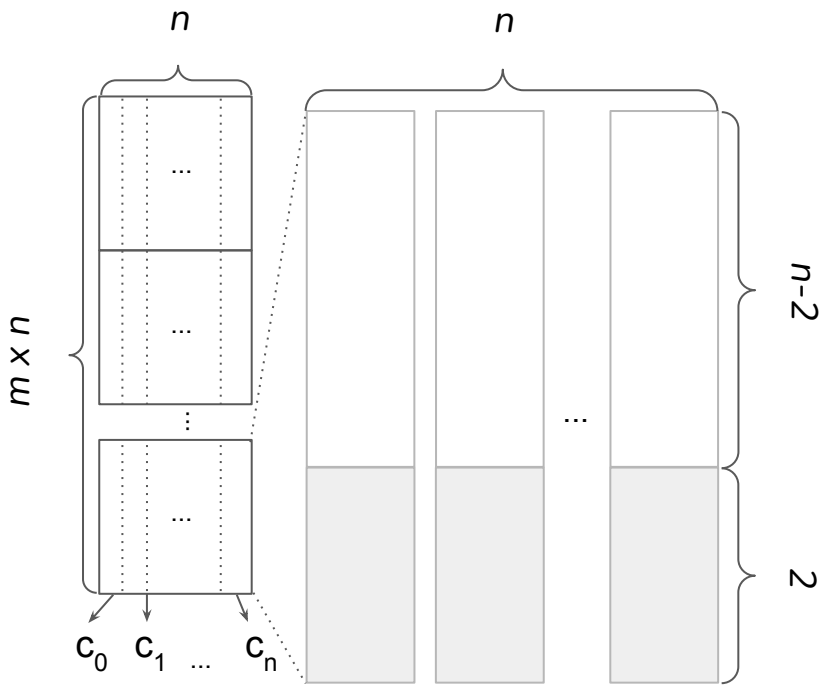
- Problem: proportion of recoverable data decreases as grid becomes larger

- Additionally, might assume we have more available servers

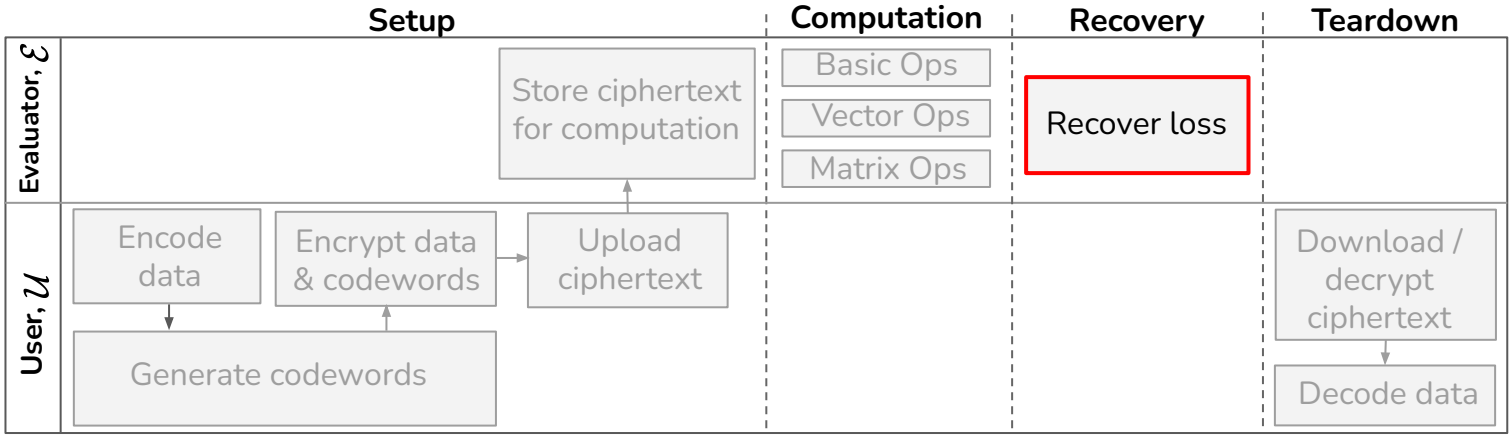


# X-Cipher Structure

- Stack *m* of the *n x n* grids vertically
- Use encoding and ciphertext packing to encrypt each column into a single ciphertext
- Perform operations on each column
- Maximizes the “utilization” of ciphertext packing

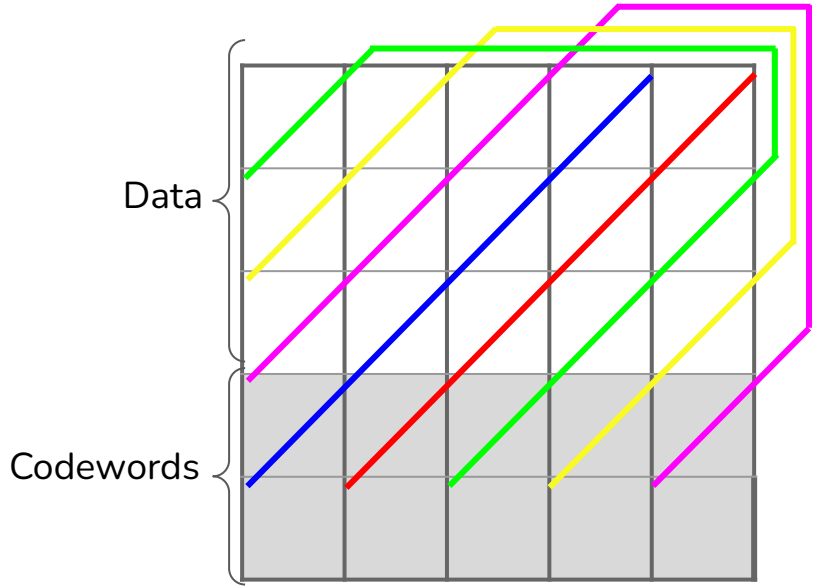


# X-Cipher Workflow



# Recovery follows algorithm of X-Code

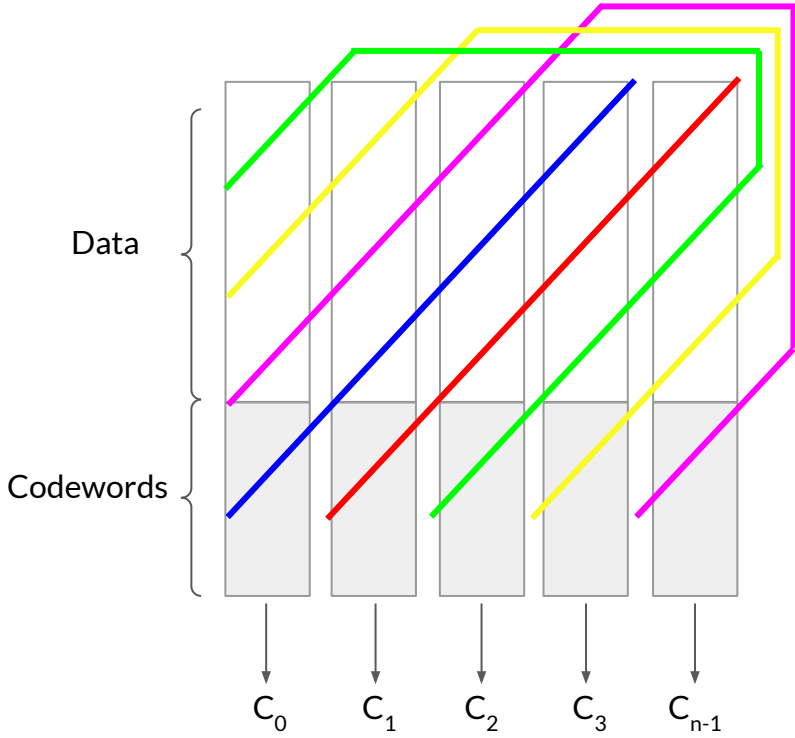
- Summing along the diagonals to recover data




Example: n=5

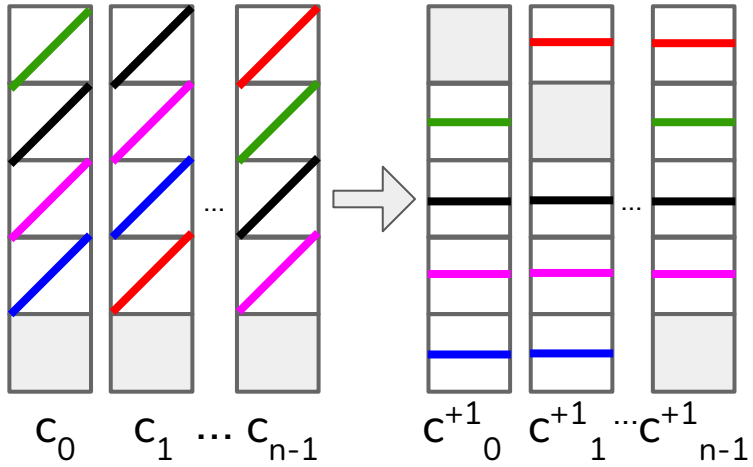
# Recovery follows algorithm of X-Code

- Summing along the diagonals to recover data
- However, ciphertexts correspond to a single column
- We must *rotate* the columns



# Ciphertext Rotation

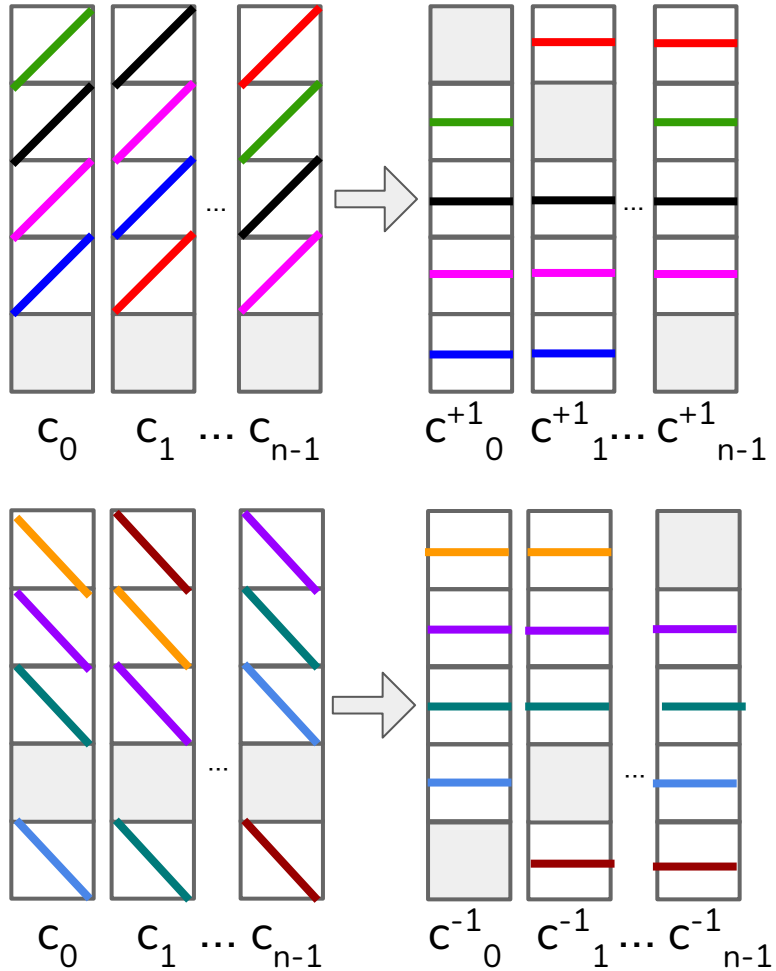
- For the +1 sloped diagonals, the column $c_i$ must be rotated by *n-i-1*



# Ciphertext Rotation

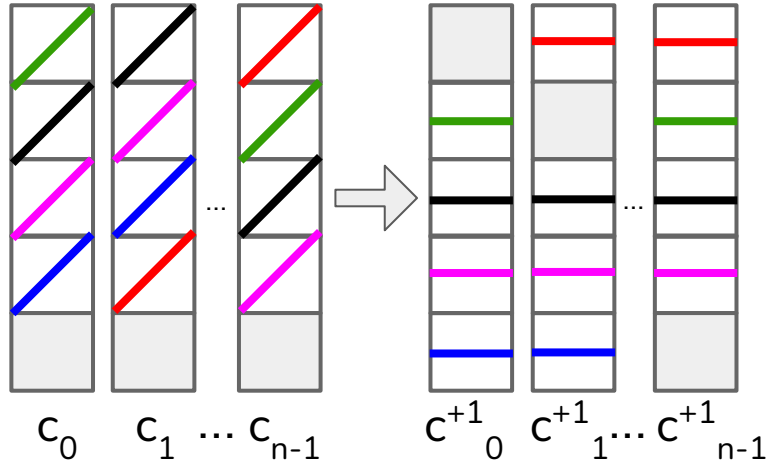
- For the +1 sloped diagonals, the column $c_i$ must be rotated by *n-i-1*

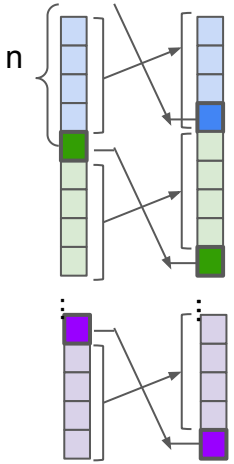
- For the -1 sloped diagonals, the column $c_i$ must be rotated by *i*



# Ciphertext Rotation

- For the +1 sloped diagonals, the column $c_i$ must be rotated by *n-i-1*
- For the -1 sloped diagonals, the column $c_i$ must be rotated by *i*
- Recall: X-Cipher structure is multiple square grids

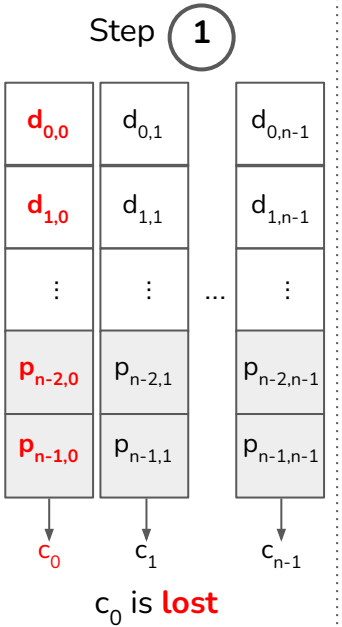



**X-Cipher rotation algorithm applies to all internal square grids simultaneously**

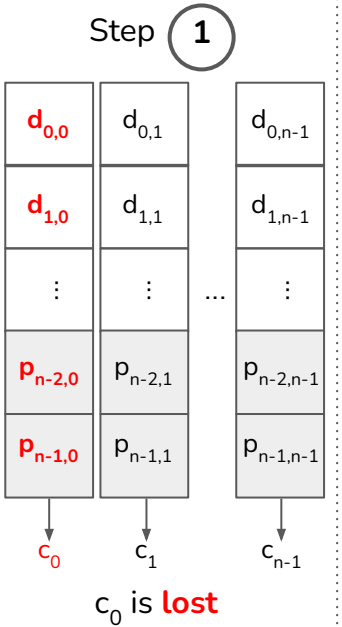
# Rotation for one column recovery

Consider a case in which Adversary has caused failure in Server 0 ($c_0$ is lost)



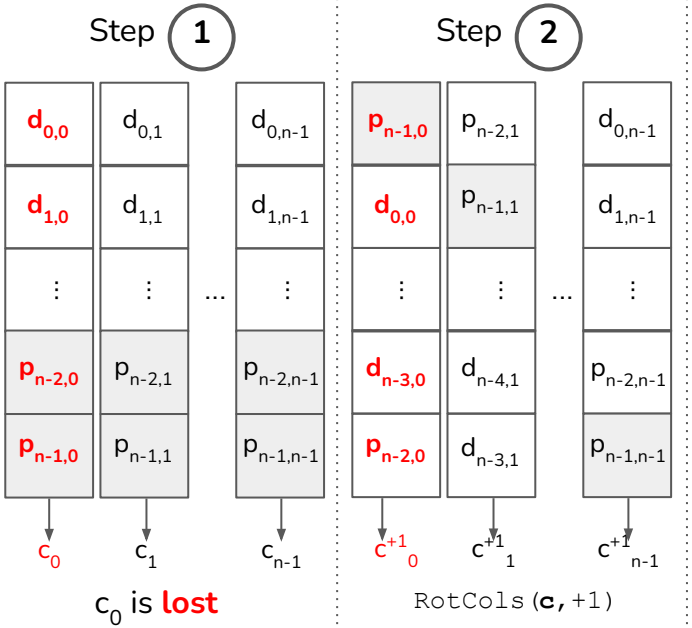
# Rotation for one column recovery

First we, detect $c_0$ is lost due to a lack of response from Server 0



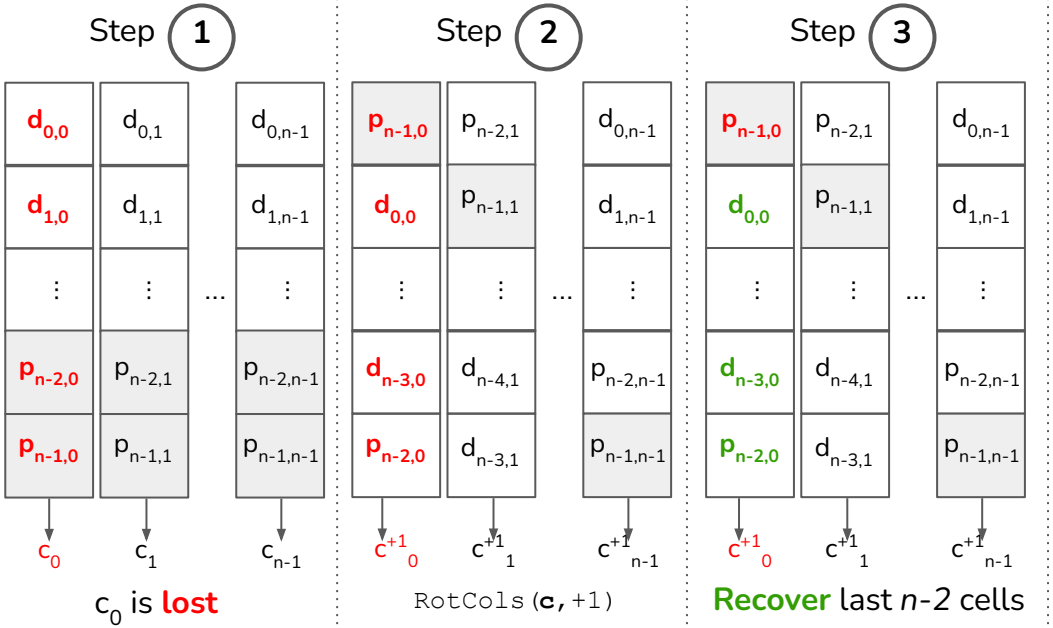
# Rotation for one column recovery

Second, we begin rotating and summing to recover via +1 sloped codewords



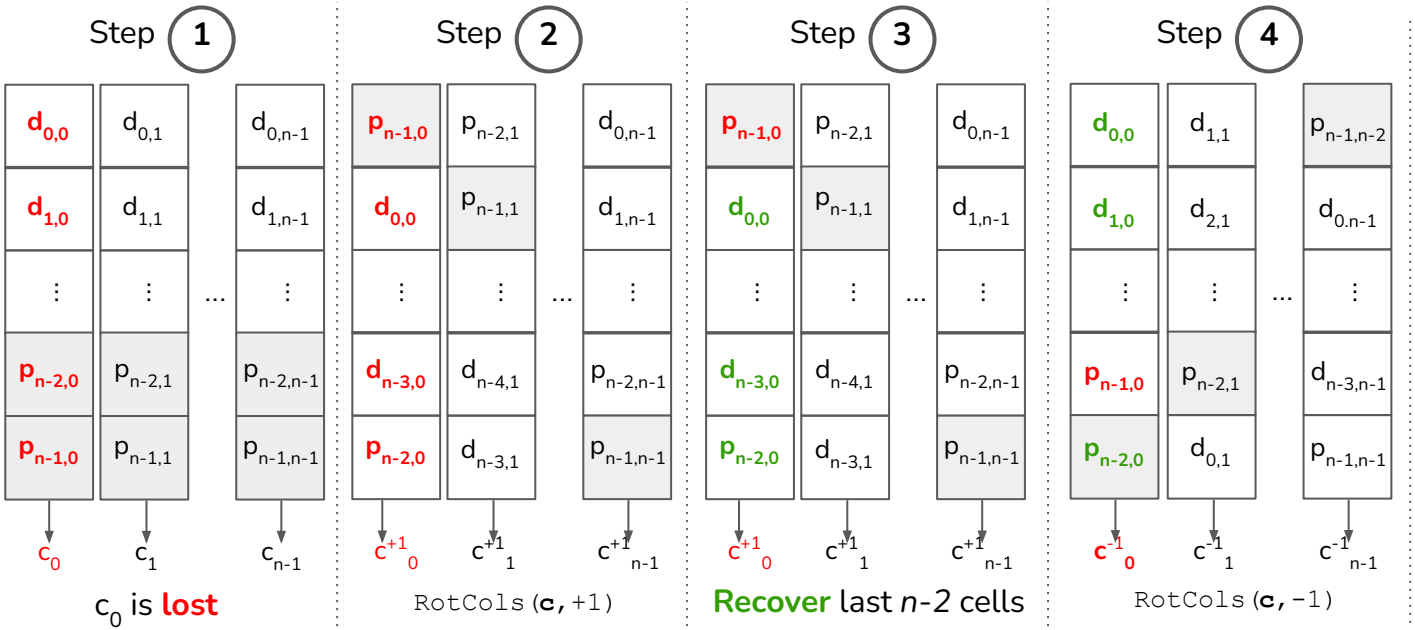
# Rotation for one column recovery

We **recover** all data within $c_0$ except for the final codeword $p_{n-1,0}$



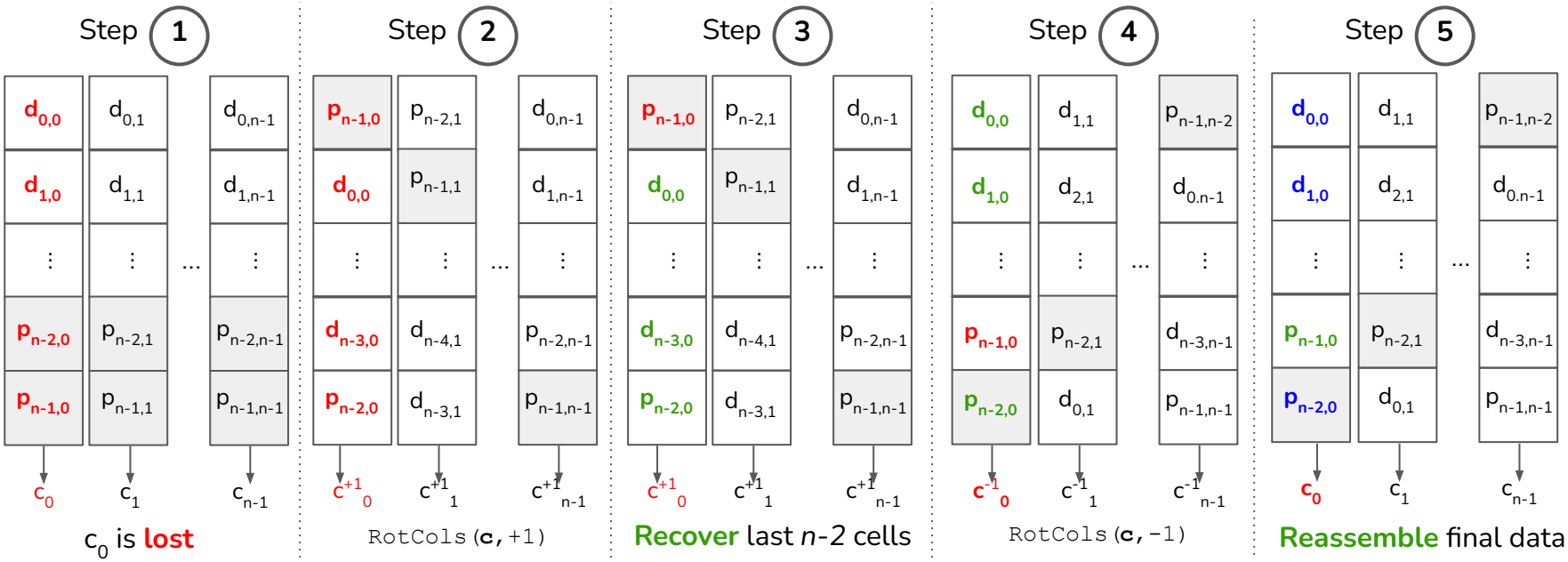
# Rotation for one column recovery

Perform another rotation and summing to recover the final data



# Rotation for one column recovery

$c_0$ has now been fully recovered. It can be redistributed to continue operations



# Other features (see paper for details!)

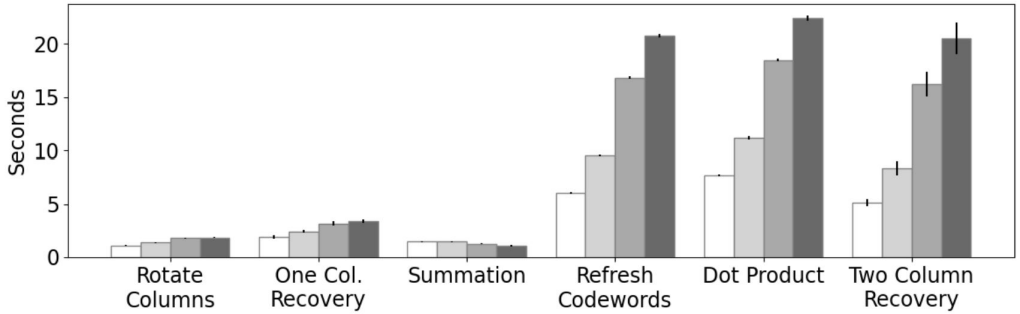
- Two column recovery → multiple iterations of the one-column recovery

- Basic operations / Primitive operations:
  - Homomorphic arithmetic, refresh codewords, dot-product, summation

- Demonstrate construction of complex algorithms:
  - Private set intersection (PSI)
  - Matrix multiplication

# Results

- Evaluated using CloudLab
- Primitive function timing
- Ciphertext size impact




| Parameters | | | Size (KB) | | |
|---|---|---|---|---|---|
| Dimension (n) | Multiples (m) | Data Cells | Plaintext | Ciphertext (X-Cipher) | Ciphertext (without) |
| 5 | 7 | 180 | 0.72 | 0.93 | 55.8 |
| 7 | 9 | 315 | 1.26 | 1.30 | 82.1 |
| 11 | 5 | 495 | 1.98 | 2.05 | 112.5 |
| 13 | 4 | 572 | 2.28 | 2.42 | 125.7 |

# Thank you!

ICICS Mytilene, Greece 2024

RIT | Rochester Institute of Te

**Information:**

Paper (via ICICS):

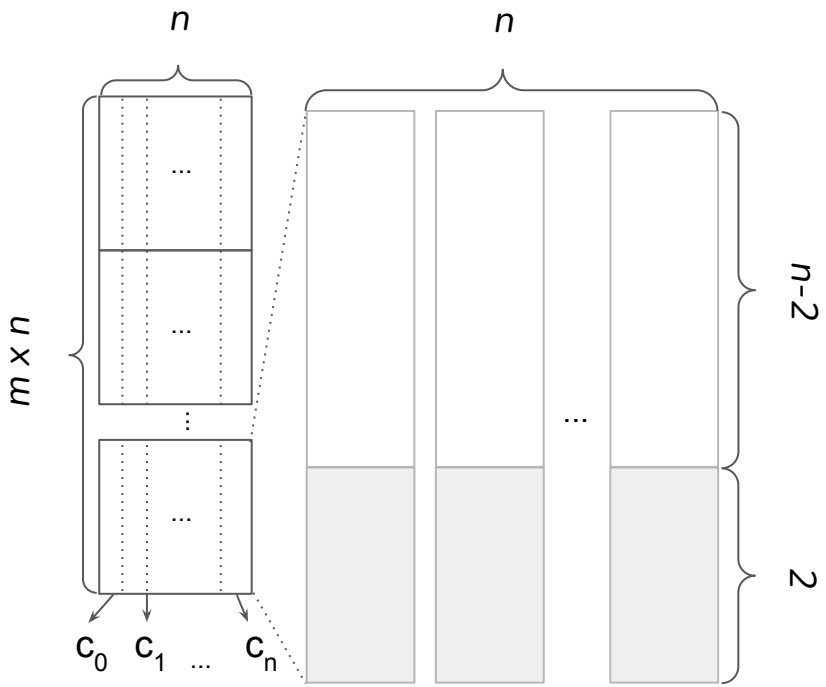
Code (via Github):

**Contact:**

My email (ac7717@rit.edu)

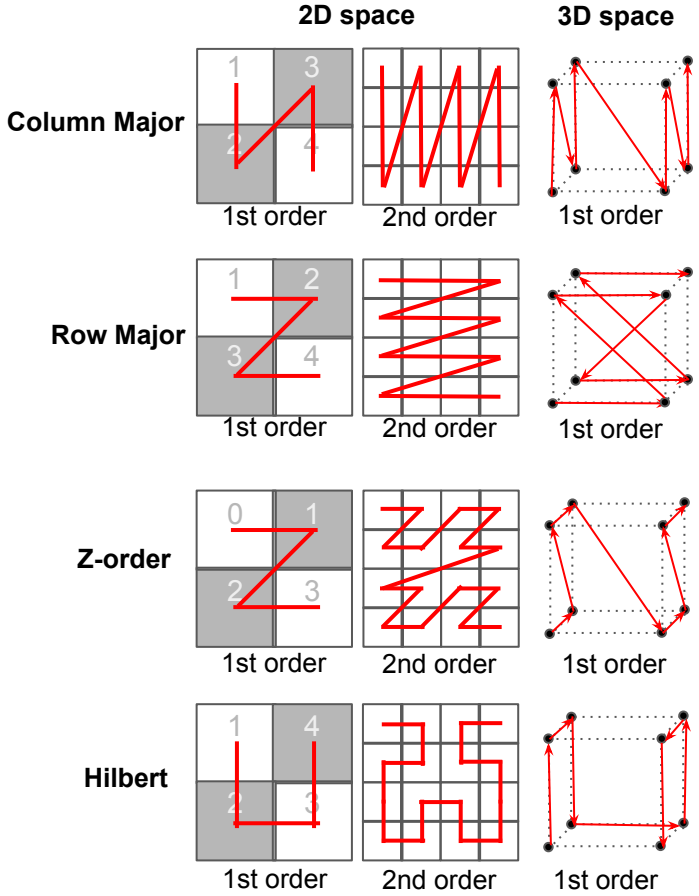
# X-Cipher Structure

- After these steps, data (plaintext and codewords) are in X-Cipher columns
- They are packed and encrypted before being distributed among servers for computations
- Set of ciphertexts: $\{c_0, c_1, \ldots c_n\}$



# X-Cipher Structure

- Data is encoded into the X-Cipher structure in column-major format
- But the information that is encoded into each column depends on the application
- Traverse any of the following ways to create the input stream into X-Cipher



# Optimized polynomial encoding: Ciphertext Packing

Instead of one plaintext per polynomial, ***pack*** a vector of plaintext elements into a single polynomial

Plaintext vector $v_A = [8, 5, 16, 9, ...]$

Method 1: Packing into coefficients:

$$A(x) = 8 + 5x + 16x^2 + 9x^3$$

Straight forward, but makes multiplication more complicated